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A COMPARISON BETWEEN USING INCOHERENT OR COHERENT SOURCES TO  
ALIGN AND TEST AN ADAPTIVE OPTICAL TELESCOPE

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## INTRODUCTION

A segmented mirror adaptive optics telescope called the phased array mirror extendable large aperture telescope (Pamela)<sup>1</sup> was designed by Kaman Aerospace Corporation, delivered to Marshall Space Flight Center, and is under study. This telescope has a spherical mirror of radius of curvature of 1.5227 m and consists of 36 segments forming a telescope 0.5 m in diameter. It is a prototype for a planned 12 m telescope having 144,000 segments. This proposed telescope will be used for laser power beaming and may also be used for imaging. A secondary mirror forming a catadioptric element corrects for spherical aberration. An additional lens system renders the emergent beam parallel. A portion of the emergent beam is deflected with a beamsplitter into an imaging CCD detector array. The remainder of the beam is sampled by a Hartmann wavefront sensor (WFS)<sup>2</sup> which has a detector element corresponding to each mirror segment and the detectors are lateral effect diodes (LEDD)<sup>3</sup>. These elements measure the tilt produced by atmospheric turbulence. In correcting for atmospheric turbulence a point source beacon is used, which may be a star or artificially produced guide star<sup>4</sup> and these are incoherent sources. The atmosphere distorts the spherical wave-front from the beacon and the amount of phase (tilt) distortion is measured using the WFS. This distortion is removed using the adaptive optics telescope and the laser power beam is transmitted after this correction. The mirror must be corrected and the laser beam must be transmitted in less than 10 ms which is the approximate atmospheric coherence time so all intensity measurements are considered instantaneous.

Additional optics are required to align the telescope and these are a Wyko interferometer for segment tilt adjustment and a white light interferometer to adjust the pitch across the segment boundaries. Finally, a spatial filtered, expanded, collimated HeNe laser beam is used after alignment to record the central position of the 36 beams in the 36 LEDDs. In test experiments an aberrator is placed in this HeNe laser arm after the collimator and the closed-looped WFS-actuator-computer control system can be tested under various distortion conditions.

The concept in the initial alignment of the telescope is to produce an optical transfer function (OTF) which closely approximates the diffraction limited value<sup>5</sup> which would correspond to a system pupil function that is unity over the aperture and zero outside<sup>6</sup>. In some experiments an additional computer calculates the OTF. When the system is aligned, the WFS is calibrated by recording the spot position on each detector element. When an atmospheric distorted wave-front is intercepted by the telescope pupil plane, the WFS measures the wave-front phase distortion by noting the change in spot positions and the segment actuator-control computer returns the spot on each WFS LEDD to the position it had for the aligned telescope by adjusting mirror segments. The atmosphere-instrument system is represented by an atmosphere-instrument OTF and the actuator-control computer system adjusts segments to remove any wave-front error and reproduce the original aligned telescope OTF.

There are differences in the theory of intensity measurements between coherent and incoherent radiation. As a result, some of the classical quantities which describe the performance of an optical system for incoherent radiation can not be defined for a coherent field. The most important quantity describing the quality of an optical system is the OTF and for a coherent source the OTF is not defined. Instead a coherent transfer function (CTF)<sup>6</sup> is defined.

## THEORY

### A. Theory For An Incoherent Source

Consider the telescope is in the alignment mode and an incoherent plane wave-front is incident on it. After alignment the telescope receives a signal from a distant incoherent point source called a beacon. In alignment the telescope is adjusted to give a system OTF which closely approximates the diffraction limited value. For a distant incoherent beacon the incident wave-front is plane and spatially coherent before distortion is produced by atmospheric turbulence. The WFS and actuator-control computer system attempt to correct the mirror to its aligned OTF. In the case of Pamela each WFS element is masked and measures the atmospheric phase (tilt) distortion for a single segment of the telescope with no overlap between segments. As a result, each sensor detector element has a pupil function which exists over the extent of the segment and goes to zero outside the segment. An OTF can be associated with each mirror segment and system has the total OTF which is the sum of these segment OTFs. The entire telescope should be corrected so this discussion will refer to the OTF of the entire telescope for this is what is sensed by the imaging CCD array.

In the alignment mode an incoherent plane wave is incident on the telescope. The pupil function of the telescope at point  $\vec{r}$  of coordinates (x,y) in the detector's pupil plane is  $h_o(\vec{r})$  and the system point spread function (PSF) is  $o_o(\vec{r})$ . The real image intensity<sup>7</sup>  $i(\vec{r})$  in the pupil plane of the detector is

$$i(\vec{r}) = o_o(\vec{r}) \otimes i_s(\vec{r}) = |h_o(\vec{r})|^2 \otimes i_s(\vec{r}) \quad (1)$$

where  $i_s(\vec{r})$  is the source or object intensity distribution projected to the real image plane and is the magnified image of the object and  $\otimes$  is the convolution operator. If the Fourier transform<sup>8</sup> is taken of equation (1), the image intensity in spatial frequency space  $\vec{u}$  is

$$I(\vec{u}) = O_o(\vec{u}) I_s(\vec{u}) = [H_o(\vec{u}) \odot H_o^*(\vec{u})] I_s(\vec{u}) \quad (2)$$

where the capital letters correspond to the Fourier transformed values of the corresponding small letters in the previous equation and  $\odot$  is the autocorrelation operation. The quantity  $I(\vec{u})$  is the real image spectrum,  $I_s(\vec{u})$  is the object spectrum,  $O_o(\vec{u})$  is the OTF of the aligned telescope,  $H_o(\vec{u})$  is the instrument pupil function spectrum. The data from the WFS are recorded after alignment and if it is desired, the data from the WFS can be Fourier transformed to yield  $I(\vec{u})$  and if the incident wave-front is plane, its amplitude is constant so  $I_s(\vec{u})$  can be measured and removed from equation (2) and the instrument OTF can be evaluated.

If an incoherent point source a large distance (effectively infinite) from the telescope is imaged through the atmosphere, the image in the pupil plane of the telescope is

$$i_a(\vec{r}) = o_a(\vec{r}) \otimes i_s(\vec{r}) = |a(\vec{r}) \otimes h_o(\vec{r})|^2 \otimes i_s(\vec{r}) \quad (3)$$

where  $i_s(\vec{r})$  is the atmospheric turbulence distorted image and  $o_a(\vec{r})$  is the atmosphere-

instrument PSF. The PSF is the absolute value squared of the product of an atmospheric turbulence term convoluted into the instrument pupil function. In Fourier space the signal is

$$I_a(\bar{u}) = O_a(\bar{u}) I_s(\bar{u}) = [A(\bar{u})H_o(\bar{u})] \odot [A(\bar{u})H_o(\bar{u})]^* I_s(\bar{u}) \quad (4)$$

where capital letters are the Fourier transformed values. The OTF is the autocorrelation of the term  $[A(\bar{u})H_o(\bar{u})]$ . In an adaptive optics system the WFS and actuator-control computer try to restore the system to the original instrument OTF,  $[H_o(\bar{u}) \odot H_o^*(\bar{u})]$ .

## B. Theory For A Coherent Source

For a laser's Gaussian beam profile a collimated beam can not be obtained using a lens system<sup>9</sup>. There are two definitions of collimation of Gaussian beams. One is the divergence angle is made as small as possible and the other is the next beam waist is made at the maximum distance from the lens system collimating the beam. This means the emerging beam from the collimator is a truncated Gaussian beam and it remains a truncated Gaussian in both the real and spatial frequency space. The HeNe laser used to align Pamela exhibits an approximate 20% fall-off at its input to the WFS. The beam is a spatially and temporally coherent truncated Gaussian function. For a coherent source the field can be calculated at any point along the beam path and the detector measures the intensity at that point. In the alignment mode for a coherent beam the field at a point  $\bar{r}$  in the pupil plane of the detector is

$$e_c(\bar{r}) = h_o(\bar{r}) \otimes e_{sc}(\bar{r}) \quad (5)$$

where  $e_c(\bar{r})$  is the calculated field in the pupil plane of the detector,  $e_{sc}(\bar{r})$  is the object field in this plane, and  $h_o(\bar{r})$  is the pupil function of the telescope in this plane. The intensity<sup>7</sup> in the pupil plane of the detector is

$$i_c(\bar{r}) = [h_o(\bar{r}) \otimes e_{sc}(\bar{r})] [h_o^*(\bar{r}) \otimes e_{sc}^*(\bar{r})]. \quad (6)$$

In the Fourier plane the intensity becomes

$$I_c(\bar{u}) = [H_o(\bar{u}) E_{sc}(\bar{u})] \odot [H_o^*(\bar{u}) E_{sc}^*(\bar{u})]. \quad (7)$$

The intensity is the autocorrelation of the product of the Fourier transformed pupil function and the spatial dependent transformed field and it is not the autocorrelation of just the transformed pupil function which is the classical definition of the OTF. If the field in the real plane at each WFS element exhibits a measurable lateral spatial coordinate dependence (coordinates perpendicular to the direction of propagation of the beam), the recorded calibration position for the aligned telescope will be in error. If at the CCD the Fourier transformed field exhibits spatial frequency dependence, this dependence can not be removed from the autocorrelations in equation (7) and the instrument OTF can not be correctly evaluated.

A laser source beacon at a large distance from the telescope has been proposed to phase-up the adaptive system for the effects of atmospheric turbulence. Assume the coherent laser is

operating in the TEM<sub>00</sub> mode to simplify the theory<sup>10</sup>, then the radial or lateral spatial dependence of the laser field is

$$\begin{aligned} E(r) &= E_o \omega_o / \omega(z) \exp\{-r^2[1/\omega^2(z) + jk/2R(z)]\}, \\ \omega^2(z) &= \omega_o^2[1 + z^2/z_o^2], R(z) = z[1 + z_o^2/z^2], \text{ and } z_o = \pi\omega_o^2 n/\lambda \end{aligned} \quad (8)$$

where  $\lambda$  is the laser wavelength and  $\omega_o$  is the beam waist. The range is the distance from the beacon to the telescope and if  $z = R$  goes to infinity, then the field at the telescope is independent of  $r$  so the field at the telescope is approximately

$$E(R) \approx E_o \omega_o^2 / \lambda R \exp[-jkr^2/2R] \approx E_o \omega_o^2 / \lambda R. \quad (9)$$

The transformed fields are also independent of the coordinates  $\vec{r}$  and  $\vec{u}$  in the real and Fourier planes.

In the case of the distant laser beacon traversing the atmosphere the field in the pupil plane of the detector is

$$e_c(\vec{r}) = a(\vec{r}) \otimes h_o(\vec{r}) \otimes e_{sc}(\vec{r}). \quad (10)$$

The intensity measured by the detector is

$$i_c(\vec{r}) = [a(\vec{r}) \otimes h_o(\vec{r}) \otimes e_{sc}(\vec{r})] [a^*(\vec{r}) \otimes h_o^*(\vec{r}) \otimes e_{sc}^*(\vec{r})]. \quad (11)$$

The intensity in spatial frequency space is

$$I_c(\vec{u}) = [A(\vec{u})H_o(\vec{u})E_{sc}(\vec{u})] \odot [A^*(\vec{u})H_o^*(\vec{u})E_{sc}^*(\vec{u})] \quad (12)$$

but since the distant laser field can be assumed independent of the lateral position coordinate  $\vec{u}$ , the field may be removed from the autocorrelation integrals and equation (12) has the same form as in the incoherent light case or

$$I_s(\vec{u}) = [A(\vec{u})H_o(\vec{u})] \odot [A^*(\vec{u})H_o^*(\vec{u})] I_{sc}. \quad (13)$$

## CONCLUSIONS

The main conclusion of the paper is that an incoherent collimated source and not a collimated laser source is preferred to calibrate the WFS of an aligned adaptive optical system. A distant laser source can be used with minimum problems to correct the system for atmospheric turbulence. The collimation of the HeNe laser alignment source can be improved by using a very small pin hole in the spatial filter so only the central portion of the beam is transmitted and the beam from the filter is nearly constant in amplitude. The size of this pin hole will be limited by the sensitivity of the LEDD elements.

## REFERENCES

1. Phased Array Mirror Extendable Large Aperture Telescope, Phase III, Pamela Program, Kaman Aerospace Corporation, Contract # DASG 60-90-0022, 1993.
2. Lane, R.G. and Tallon, M., Wave-Front Reconstruction Using a Shack-Hartmann Sensor, *Applied Optics*, Vol. 31, pp 6902-6907, 1992.
3. Kelley, B.O. and Nemhauser, R.I., Technique for Using the Position Sensitivity of Silicon Photodetectors to Provide Remote Machine Control, 21st Annual IEEE Machine Tool Conference, paper C73970-1IA, Hartford, Conn., October 29-31, 1973.
4. Happer, W., MacDonald, G.J., Max, C.E., and Dyson, F.J., Atmospheric-Turbulence Compensation by Resonant Optical Backscattering from the Sodium Layer in the Upper Atmosphere, *Journal of the Optical Society of America*, Vol. A11, pp 263-276, 1994.
5. Roggemann, M.C. and Meinhardt, J.A., Image Reconstruction by Means of Wave-Front Sensor Measurement in a Closed-Loop Adaptive-Optics Systems. *Journal of the Optical Society of America*, Vol. A10, pp 1996-2007, 1993.
6. Goodman, J.W., *Introduction to Fourier Optics*, McGraw-Hill, ISBN 0-07-023776-X, pp 111-115, 1968.
7. Mavroidis, T., Dainty, J.C., and Northcott, M.J., Imaging Coherently Illuminated Objects Through Turbulence: Plane-Wave Illumination, *Journal of the Optical Society of America*, Vol. A7, pp 348-355, 1990.
8. Gaskill, J.D., *Linear Systems, Fourier Transforms, and Optics*, John Wiley & Sons, ISBN 0-471-29288-5, pp 313-317, 1978.
9. O'Shea, D.C., *Elements of Modern Optical Design*, John Wiley & Sons, ISBN 0-471-07796-8, pp241-245, 1985.
10. Yariv, A., *Optical Electronics*, Saunders College Publishing, ISBN 0-03-04744-2, pp 47-48, 1991.